MUSTAFA KEMAL ÜNİVERSİTESİ

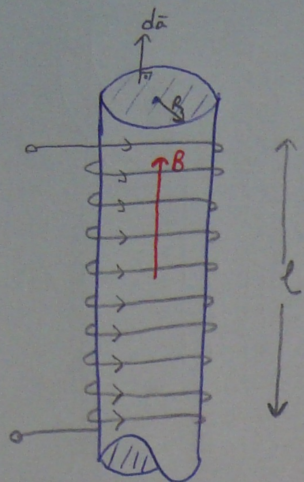
MAĞNETİK SORULARI VE ÇÖZÜMLERİ

Sarper ÖNAL
“Herkes geçsin diye voll 2.0”
doya doya geçin!

Magnetik II. Vizeye Hazırlık Soruları

SONAL
14.05.2010

Soru 1: $L = ?$ $\quad w_m = ?$



Çok uzun selenoid

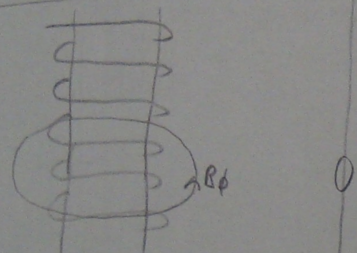
$$B = \mu_0 I N' (i_z) \quad ; \quad N' = \frac{N}{\ell}$$

$$\phi_n = \phi = \iint \bar{B} \cdot d\bar{a} = \mu_0 I N' (\pi R^2)$$

$$\boxed{\phi = \frac{\mu_0 I N \pi R^2}{\ell}}$$

$$L_{11} = L_1 = \frac{N \cdot \phi}{I} = \frac{\mu_0 \cancel{I} \cdot N^2 \pi R^2 \cdot \ell}{\cancel{I} \cdot \ell \cdot \ell} = \mu_0 (N')^2 \cdot \pi R^2 \cdot \ell \text{ Henry}$$

$$\boxed{L_1 = \mu_0 (N')^2 \pi R^2 \ell \text{ Henry}} \qquad \boxed{\frac{L_1}{\ell} = \mu_0 (N')^2 \pi R^2 \ \frac{\text{Henry}}{\text{metre}}}$$



Selenoidde Bφ dikkate alınmaz çok küçüktür

$$W_m = \frac{1}{2} L_1 I_1^2 = \frac{1}{2} \cdot \mu_0 (N')^2 \pi R^2 \ell \cdot I^2 = \frac{B^2 \cdot (N')^2 \cdot \pi R^2 \ell}{2 \mu_0}$$

$$\boxed{L_1 = \frac{1}{\mu_0 I^2} \cdot \int B^2 \, dV.}$$

Soru 2:

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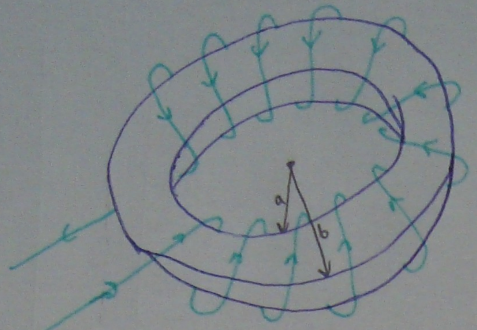
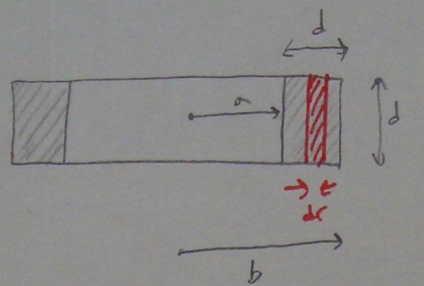


N sarımlı .... L'sini bulun

Çözüm:



$da = d . dr$

$$\oint B . dl = \mu_0 I = \mu_0 N I$$

$(2\pi r) B$

$$\boxed{B = \frac{\mu_0 I N}{2\pi r} i_\phi}$$

$$\phi = \iint B . da = \int_a^b \frac{\mu_0 I N}{2\pi r} . d\, dr$$

$$= \frac{\mu_0 I N d}{2\pi} \int_a^b \frac{dr}{r} = \boxed{\frac{\mu_0 I N d}{2\pi} . \ln\frac{b}{a}}$$

$$L = \frac{N\phi}{I} = \boxed{\frac{\mu_0 N^2 d}{2\pi} . \ln\left(\frac{b}{a}\right) \text{ Henry}}$$

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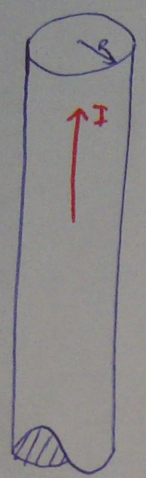
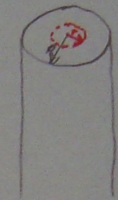
Soru3:



$r<R$ için ve $r>R$ için ϕ'yi bulunuz.

Çözüm:

$r<R$ için:

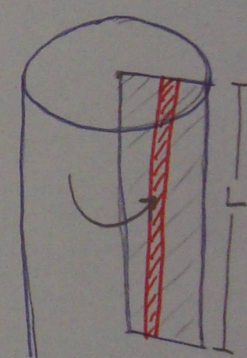


$$I = J\cdot\pi R^2 \quad ; \quad J=\frac{I}{\pi R^2}$$

$$\oint B\,dl = \mu_0 I_{net}$$

$$2\pi r B = \mu_0\cdot J\pi r^2 = \mu_0 \frac{I}{\pi R^2}\cdot \pi r^2$$

$$B=\frac{\mu_0 I r^2}{2\pi r R^2} \Rightarrow \boxed{B=\frac{\mu_0 I r}{2\pi R^2}\, i_\phi}$$



$$\phi_{iç} = \iint B'\cdot da \quad ; \quad B' = \frac{\pi r^2}{\pi R^2}\cdot B = \frac{r^2}{R^2}B$$

$$\phi_{iç} = \int_0^R \frac{r^2}{R^2}\cdot\frac{\mu_0 I r}{2\pi R^2}\cdot L\,dr = \frac{\mu_0 I L}{2\pi R^4}\int_0^R r^3 dr = \frac{\mu_0 I L}{2\pi R^4}\cdot\frac{r^4}{4}\Big|_0^R = \frac{\mu_0 I L R^4}{2\pi R^4\cdot 4}$$

$$\boxed{\phi_{iç} = \frac{\mu_0 I L}{8\pi}}$$

$r>R$ için:

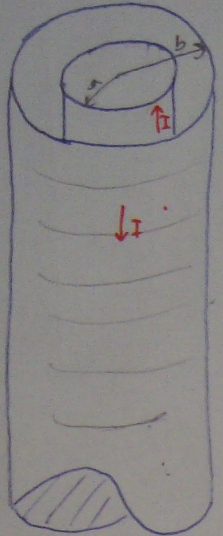
$r>R$ için $\phi = ?$

?

Sonuç
14.05.2010

Soru 4

Her yerde L'yi bulunuz



Çözüm:

r<a için:

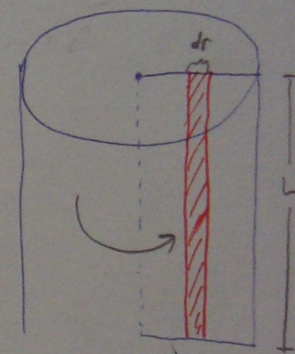
$I = J.\pi a^2$; $J = \frac{I}{\pi R^2}$

$\oint B dl = \mu_0 I_{net}$

$2\pi r B = \mu_0 J \pi r^2 = \mu_0 \frac{I}{\cancel{\pi} R^2} . \cancel{\pi} r^2$

$B = \frac{\mu_0 I r^2}{2\pi r R^2} \Rightarrow \boxed{B = \frac{\mu_0 I r}{2\pi R^2} \, i_\phi}$

$da = L.dr$



$\phi_{iç} = \iint B' . da$; $B' = \frac{\pi r^2}{\pi R^2} . B = \frac{r^2}{R^2} B$

$\phi_{iç} = \int_0^R \frac{r^2}{R^2} . \frac{\mu_0 I r}{2\pi R^2} . L . dr = \frac{\mu_0 I L}{2\pi R^4} \int_0^R r^3 dr = \frac{\mu_0 I L r^4}{8\pi R^4} \Big|_0^R$

$\boxed{\phi_{iç} = \frac{\mu_0 I L}{8\pi}}$; $L_{iç} = \frac{\phi_{iç}}{I} = \frac{\mu_0 L}{8\pi}$ Henry

$\frac{L_{iç}}{L} = \frac{\mu_0}{8\pi} = \frac{4\pi \times 10^{-7}}{8\pi} = \boxed{0,5 \times 10^{-7} \, \frac{H}{m}}$

a<r<b için:

$\oint B dl = \mu_0 I$

$2\pi r B = \mu_0 I$

$\boxed{B = \frac{\mu_0 I}{2\pi r} \, i_\phi}$

$\phi = \iint B da = \int_a^b B . L dr = \int_a^b \frac{\mu_0 I}{2\pi r} . L . dr = \boxed{\frac{\mu_0 I L}{2\pi} . \ln \frac{b}{a}}$

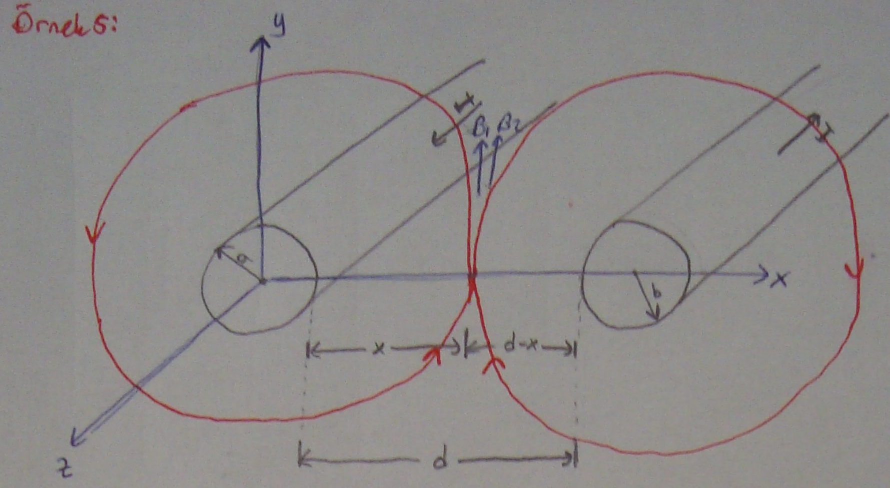
$L_{dış} = \frac{\phi}{I} = \frac{\mu_0 L}{2\pi} \ln \frac{b}{a}$

$\boxed{\frac{L_{dış}}{L} = \frac{\mu_0}{2\pi} \ln \frac{b}{a}}$

r>b için: $r > b \Rightarrow B = 0$ // $(I_{net} = I - I = 0)$

$L_{Toplam} = L_{iç} + L_{dış} = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) = \boxed{\frac{\mu_0}{2\pi} . \left(\frac{1}{4} + \ln\left(\frac{b}{a}\right)\right) \frac{H}{m}}$ //

Örnek 6:



Heryerde L yi bulun

$$B_1 = \frac{\mu_0 I}{2\pi x}(i_y) \quad ; \quad B_2 = \frac{\mu_0 I}{2\pi (d-x)}(i_y) \quad ; \quad d\vec{a} = l.dr$$

Liç için:

$$\oint B\,dl = \mu_0 I_{net} \quad ; \quad J = \frac{I}{\pi R^2} \qquad B = \frac{\mu_0 I r^2}{2\pi r a^2} = \frac{\mu_0 I r}{2\pi a^2}$$

$$2\pi r B = \mu_0 J . \pi r^2 = \mu_0 \frac{I}{\pi a^2} . \pi r^2$$

$$\phi_{iç} = \iint B' . da \quad ; \quad B' = \frac{\pi r^2}{\pi a^2} . B = \frac{r^2}{a^2} B$$

$$\phi_{iç} = \int_0^a \frac{r^2}{a^2} . \frac{\mu_0 I r}{2\pi a^2} . l . dr = \frac{\mu_0 I l}{8\pi a^4} . \frac{r^4}{4}\Big|_0^a = \frac{\mu_0 I l}{8\pi}$$

$$L_{iç} = \frac{\phi_{iç}}{I} = \frac{\mu_0 l}{8\pi} \text{ Henry} \quad ; \quad \boxed{\frac{L_{iç}}{l} = \frac{\mu_0}{8\pi} \frac{\text{Henry}}{\text{metre}}}$$

Ldış için: $d\vec{a} = l\,dx$

$$\phi = \iint B\,d\vec{a} = \int_a^{d-a} (B_1 + B_2) . da = \frac{\mu_0 I l}{2\pi} . \int_a^{d-a} \left(\frac{1}{x} + \frac{1}{d-x}\right) dx = \frac{\mu_0 I l}{2\pi} \left[\underbrace{\left[\ln x - \ln (d-x)\right]\Big|_a^{d-a}}_{\left(2 \ln \frac{d-a}{a}\right)} \right]$$

$$\phi_{dış} = \frac{\mu_0 I l}{\pi} . \ln\left(\frac{d-a}{a}\right)$$

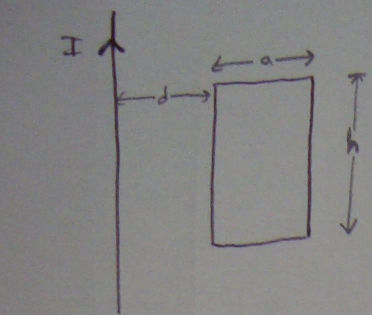
$$L_{dış} = \frac{\phi}{I} = \frac{\mu_0 l}{\pi} \ln\left(\frac{d-a}{a}\right)$$

$$\frac{L_{dış}}{l} = \frac{\mu_0}{\pi} \ln\left(\frac{d-a}{a}\right) \frac{H}{m}$$

$$L_{Top} = 2L_{iç} + L_{dış}$$

$$\boxed{\frac{L_{Top}}{l} = \frac{\mu_0}{\pi} \left[\frac{1}{4} + \ln\left(\frac{d-a}{a}\right)\right] \frac{H}{m}}$$

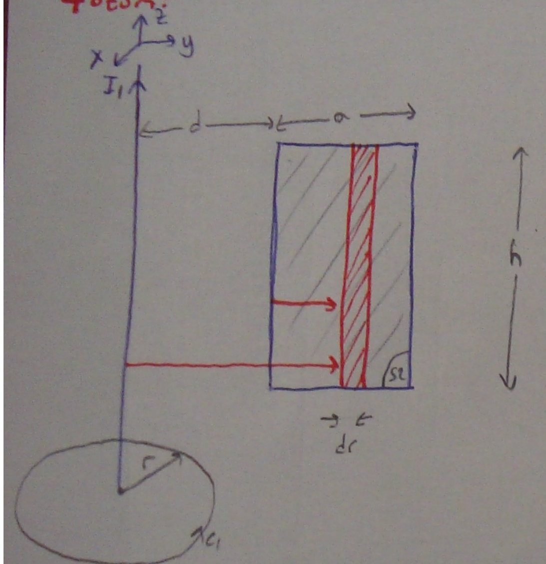
Örnek 6:



Çok uzun iletken

Karşılıklı endüktansı bulunuz.

Çözüm:



$$\Phi_{12} = \iint_{S_2} B_1 \, da_2 = \int_d^{d+a} \frac{\mu_0 I_1}{2\pi r} \, . h \, dr = \frac{\mu_0 I_1 h}{2\pi} Ln(r)\Big|_d^{d+a}$$

$$\Phi_{12} = \frac{\mu_0 I_1 h}{2\pi} . Ln\left(\frac{d+a}{a}\right)$$

$$\boxed{M_{12} = \frac{N_2 \Phi_{12}}{I_1} = \frac{N_2 . \mu_0 h}{2\pi} . Ln\left(\frac{d+a}{a}\right) \quad H}$$

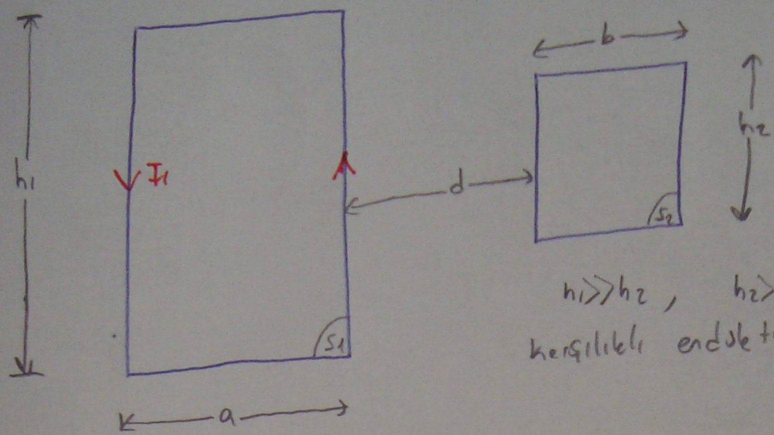
$\oint B \, dl = \mu_0 I$

$2\pi r B = \mu_0 I$

$\boxed{B = \frac{\mu_0 I}{2\pi r} i_\phi}$

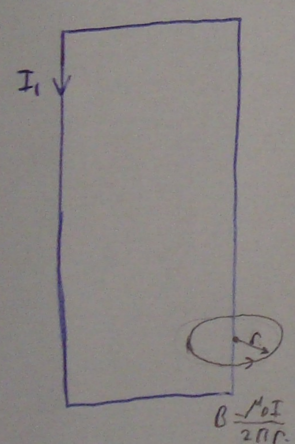
$da_2 = ds = h . dr$

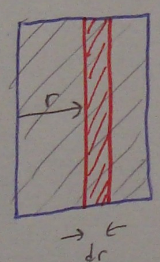
Örnek 7:



$h_1 >> h_2$, $h_2 > b > d$ bu durumda karşılıklı endüktansı bulunuz.

Çözüm)





h_1 çok çok büyükse

$\boxed{da = h_2 . dr}$

$$\phi_{12} = \iint_{S_2} B_1 da_2 = \int_0^b \left(\frac{\mu_0 I_1}{2\pi (d+r)} - \frac{\mu_0 I_1}{2\pi (d+a+r)} \right) h_2 . dr$$

$$= \frac{\mu_0 I_1 h_2}{2\pi} \int_0^b \left(\frac{1}{dr} - \frac{1}{d+a+r} \right) dr$$

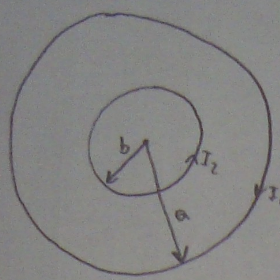
$$= \frac{\mu_0 I h_2}{2\pi} \left[\left\{ Ln(d+r) - Ln(d+a+r) \right\} \Big|_{r=0}^{b} \right]$$

$$\phi_{12} = \frac{\mu_0 I h_2}{2\pi} Ln \left[\frac{(d+b).(a+d)}{d.(a+b+d)} \right]$$

→ Eger 2. bobin N_2 sarımlı ise:

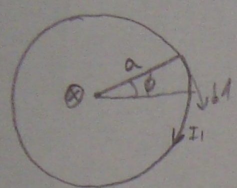
$$M_{12} = \frac{N_2 \phi_{12}}{I_1} = \boxed{\frac{\mu_0 N_2 h_2}{2\pi} Ln \left[\frac{(d+b)\,(d+a)}{d.(a+b+d)} \right]}$$

Örnek 8:



iki eşmerkezli çember ve $a \gg b$ iken karşılıklı endüktansı bulunuz.

Çözüm: önce "a" bobininin ürettiği "b"'yi kesen ϕ'yi bulalım



$$dB = \frac{\mu_0 I}{4\pi} \cdot \frac{dl \times i_r}{r^2} \quad ; \quad dl \times i_r = dl \sin 90 = a\,d\phi$$

$$dB = \frac{\mu_0 I}{4\pi} \cdot \frac{a\,d\phi}{a^2} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\phi}{a}$$

$$B = \frac{\mu_0 I}{4\pi \cdot a} \cdot 2\pi = \frac{\mu_0 I}{2a} \otimes$$

$$\boxed{B_a = \frac{\mu_0 I}{2a} \otimes}$$

Na sarımlı olan "a" çemberi için durum şöyle değişir.

$$\boxed{B_a = \frac{N_a \cdot \mu_0 I_1}{2a} \otimes}$$

$$\phi_{ab} = \iint \bar{B}_a \cdot d\bar{a}_b = \frac{N_a \cdot \mu_0 I_1}{2a} \cdot \int_0^b \int_0^{2\pi} b\,db\,d\phi = \frac{\mu_0 N_a I_1}{2a} \cdot b^2 \pi \qquad da_b = b\,db\,d\phi$$

eğer b bobini Nb sarımlı ise

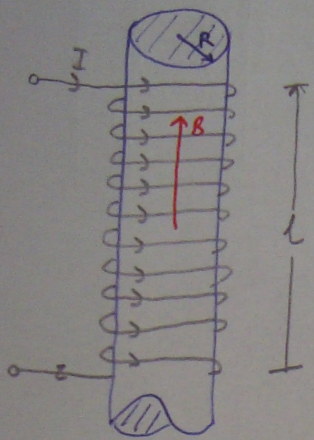
Λ_{ab}

$$M_{ab} = \frac{N_b \phi_{ab}}{I_1}$$

$$\boxed{M_{ab} = \frac{\mu_0 N_a N_b \pi b^2}{2a}}$$

Örnek 9: a) Çook uzun Selenoidin L sini bulun

b) Enerji yoğunluğunu bulun.



$B = \mu_0 I N'$ $\qquad N' = \frac{N}{l}$ $\qquad da = R\,dr\,d\phi$

$$\phi = \iint \bar{B}\cdot d\bar{a} = \iint \mu_0 I N'\, da = \mu_0 I N' \int_0^R \int_0^{2\pi} R\,dr\,d\phi$$

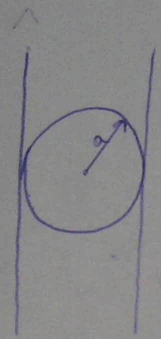
$$\phi = \mu_0 I \frac{N}{l} \cdot (R^2 \pi)$$

$$L_{11} = L_1 = \frac{N.\phi}{I} = \frac{\mu_0 \cancel{I} N (\pi R^2).N}{\cancel{I}\, l} = \frac{\mu_0 N^2 (\pi R^2)}{l\ .l} .l$$

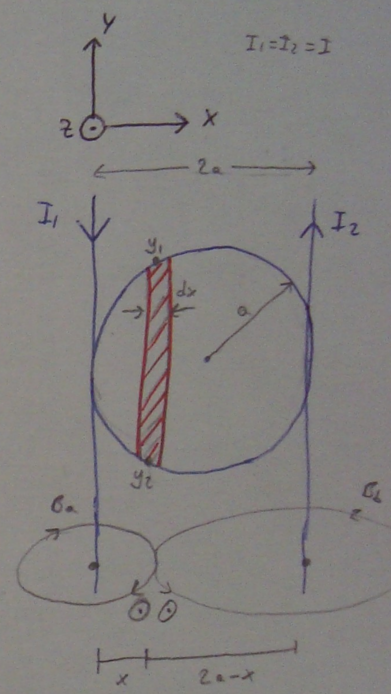
a) $\boxed{\frac{L_1}{l} = \mu_0 (N')^2 (\pi R^2) \ \frac{\text{Henry}}{\text{metre}}}$

b) $W_m = \frac{1}{2} L_1 . I_1^2 = \frac{1}{2} . \mu_0 (N')^2 . \pi R^2 . l . I^2 = \boxed{\frac{B^2 . \pi R^2 l}{2\mu_0}}$

Örnek 10:



Karşılıklı endüktansı bulun.



Çözüm:

$$B_a = \frac{\mu_0 I}{2\pi x}\, i_z \quad ; \quad B_b = \frac{\mu_0 . I}{2\pi(2a-x)}\, i_z$$

$$B_T = \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} + \frac{1}{2a-x}\right] i_z = \frac{\mu_0 I}{2\pi} \cdot \frac{2a-x+x}{2ax-x^2} \cdot i_z \ \frac{wb}{m^2}$$

$$\boxed{B_T = \frac{\mu_0 I}{\pi} \cdot \frac{a}{2ax-x^2}\, i_z \ \frac{wb}{m^2}}$$

$$\phi = \int B\, da \quad ; \quad da = (y_1 - y_2) . dx$$

Çember denklemi,
x ekseninde a noktasına kaydırılırsa ise

$$(x-a)^2 + y^2 = a^2 \Rightarrow y^2 = a^2 - (x-a)^2$$

$$y_1 = \sqrt{a^2-(x-a)^2}$$

$$y_2 = -\sqrt{a^2-(x-a)^2}$$

$$da = 2\sqrt{a^2-(x-a)^2}\, dx\, i_z$$

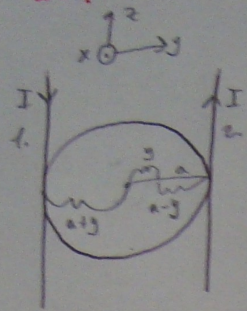
$$d\phi = \frac{\mu_0 I}{\pi} \cdot \frac{a}{2ax-x^2} \cdot 2\sqrt{a^2-(x-a)^2}\, dx$$

$$\phi = \frac{\mu_0 I 2a}{\pi}\int_0^{2a} \frac{\sqrt{a^2-x^2-a^2+2xa}}{2ax-x^2}\, dx = \frac{\mu_0 I 2a}{\pi}\int_0^{2a}\frac{\sqrt{2xa-x^2}}{2xa-x^2}\, dx = \frac{\mu_0 I 2a}{\pi}\int_0^{2a}\frac{1}{\sqrt{2xa-x^2}}\, dx$$

$$\phi = \frac{\mu_0 I 2a}{\pi}\int_0^{2a}\frac{1}{\sqrt{a^2-(x-a)^2}}\, dx \Rightarrow \quad x - a = a\sin\theta \qquad dx = a\cos\theta\, d\theta$$

$$\phi = \frac{\mu_0 I 2a}{\pi}\int_{-\pi/2}^{\pi/2}\frac{a . \cos\theta\, d\theta}{a \cos\theta} = \frac{\mu_0 I 2a}{\pi}\left(\theta\right|_{-\pi/2}^{\pi/2} = \underline{2\mu_0 I a \text{ weber}}$$

II. çözüm:



Amper kuralından

$$\oint B_1 dl_1 = \mu_0 I$$

$$B(2\pi(a-y)) = \mu_0 I$$

$$B_1 = \frac{\mu_0 I}{2\pi(a-y)} i_x$$

$$\int B_2 dl_2 = \mu_0 I$$

$$B_2(2\pi(a+y)) = \mu_0 I ,$$

$$B_2 = \frac{\mu_0 I}{2\pi(a+y)} i_x$$

Çember denklemi

$$z^2 + y^2 = a^2$$

$$z = \sqrt{a^2 - y^2}$$

$$da = 2 \cdot z \cdot dy$$

$$B_T = B_1 + B_2 = \frac{\mu_0 I}{2\pi(a-y)} + \frac{\mu_0 I}{2\pi(a+y)} = \frac{\mu_0 I a}{\pi(a^2-y^2)} i_x$$

$$\phi = \iint B \cdot da = \iint \frac{\mu_0 I a \sqrt{a^2-y^2}\, dy}{\pi \cdot (a^2-y^2)} = \frac{2\mu_0 I a}{\pi} \int \sqrt{\frac{a^2-y^2}{(a^2-y^2)^2}}\, dy$$

$$= \frac{2\mu_0 I a}{\pi} \arcsin\left(\frac{y}{a}\right)\Big|_{-a}^{a} = \frac{2\mu_0 I a}{\pi}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)$$

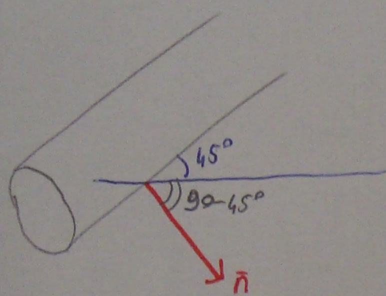
$$\phi = \frac{2\mu_0 I a}{\pi} \cdot \pi = \boxed{2\mu_0 I a \text{ weber}}$$

$$M = \frac{N_1 \phi}{I} \Rightarrow M = \frac{N_1 2\mu_0 I a}{I} = 2\mu_0 a \text{ Henry}$$

Örnek11: Magnetik alan etkisiyle indüklenen dipol moment $\bar{m} = 10^{-23}\,\frac{Am^2}{atom}$ madde içerisindeki hacim sayısı $= N = 10^{27}\,\frac{atom}{m^3}$ M ile 45° lik açı yapan yüzey üzerindeki eşdeğer yüzey akım yoğunluğu nedir?

Çözüm:

$$\bar{m} = 10^{-23}\,\frac{Am^2}{atom} \quad , \quad N = 10^{27}\,\frac{Atom}{m^3}$$

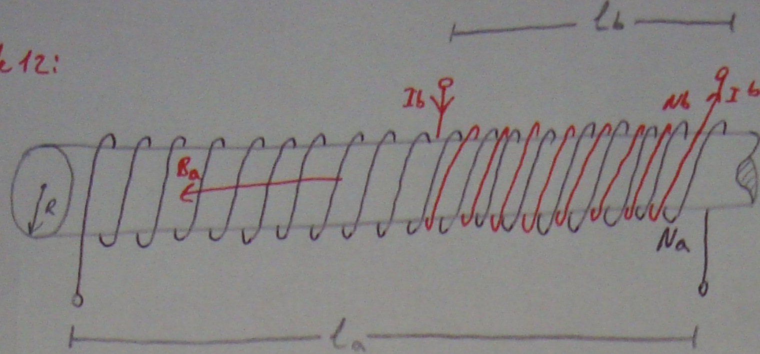


$$M = mN$$

$$\lambda_e = \bar{m} \times \bar{n} = 10^{-23} \times 10^{27} \cdot \sin(90-45)$$

$$\lambda_e = 7070\,\frac{A}{m}$$

Örnek 12:



$l_a \gg l_b$

a) Karşılıklı endüktansı bulunuz
(mutual " ")

b) $M_{ab} = M_{ba}$ olduğunu ispat ediniz

Çözüm:

$$B = \mu_0 I_a N_a' = \boxed{\mu_0 I_a \frac{N_a}{l_a}} \qquad da = r\,dr\,d\phi$$

$$\phi_{aa} = \iint B \cdot d\bar{a} = \mu_0 I_a \frac{N_a}{l_a} \int_0^R \int_0^{2\pi} r\,dr\,d\phi = \mu_0 I_a \frac{N_a}{l_a} \pi R^2 \quad ; \quad L_a = \frac{N_a \phi_{aa}}{I_a} = \boxed{\frac{\mu_0 N_a^2}{l_a} \cdot \pi R^2}$$

$$\phi_{bb} = \iint B\,da = \mu_0 I_b \frac{N_b}{l_b} \int_0^R \int_0^{2\pi} r\,dr\,d\phi = \frac{\mu_0 I_b N_b}{l_b} \pi R^2 \quad ; \quad L_b = \frac{N_b \phi_{bb}}{I_b} = \boxed{\frac{\mu_0 N_b^2}{l_b} \cdot \pi R^2}$$

$$\phi_{aa} = \phi_{ab} = \frac{\mu_0 I_a N_a}{l_a} \pi R^2 \quad ; \quad M_{ab} = \frac{N_b \cdot \phi_{ab}}{I_a} = \boxed{\frac{\mu_0 N_a N_b}{l_a} \pi R^2}$$

$$\phi_{ba} = \phi_{bb} \cdot \frac{l_b}{l_a} \quad ; \quad M_{ba} = \frac{N_a \cdot \phi_{ba}}{I_b} = \frac{\mu_0 \cancel{I_b} \cdot N_b}{\cancel{I_b}} \pi R^2 \cdot \frac{\cancel{l_b}}{l_a} \cdot \frac{N_a}{\cancel{l_b}}$$

$$\boxed{M_{ba} = \frac{\mu_0 N_a N_b}{l_a} \pi R^2}$$

$$\boxed{M_{ab} = M_{ba}}$$