MUSTAFA KEMAL ÜNİVERSİTESİ

MAĞNETİK SORULARI ve ÇÖZÜMLERİ

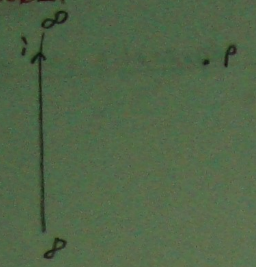
Sarper ÖNAL

"Herkes geçsin diye" vol1.9

doya doya geçin!

Sorular anlatım amaçlı çok ayrıntılı çözülmüştür.
Sınavda bukadar ayrıntılı yazmanıza gerek yok.

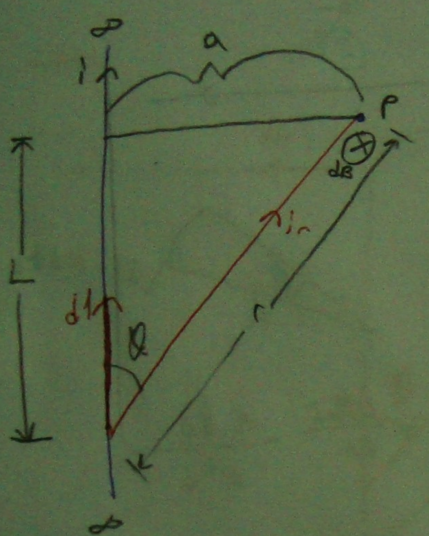
Örnek2:



P noktasındaki Magnetik akı yoğunluğu vektörünü bulun.

$$\boxed{\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}}$$

Çözüm



$|dl \times i_r| = dl \times \sin\theta$; $\sin\theta = \frac{a}{r}$; $r = \sqrt{a^2+L^2}$

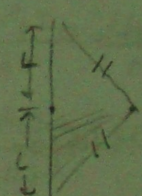
$$dB = \frac{\mu_0 I}{4\pi} \cdot \frac{dl \times i_r}{r^2} = \frac{\mu_0 I}{4\pi} \frac{dl \sin\theta}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \frac{a\,dl}{r^3} = \frac{\mu_0 I}{4\pi} \cdot \frac{a\,dl}{(a^2+L^2)^{3/2}}$$

$$dB = \frac{\mu_0 I}{4\pi} \frac{a\,dl}{(a^2+L^2)^{3/2}}$$

Simetriden dolayı

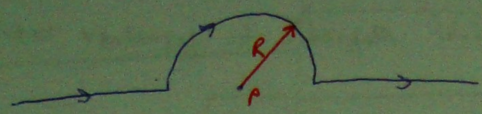
$$B = \frac{\mu_0 \cdot I \cdot a}{4\pi} 2 \cdot \int_0^\infty \frac{dl}{(a^2+L^2)^{3/2}}$$



$$B = \frac{\mu_0 I a}{2\pi} \frac{L}{a^2\sqrt{a^2+L^2}}\Bigg|_0^\infty = \frac{\mu_0 I}{2\pi a} \cdot \frac{1}{\sqrt{\left(\frac{a}{L}\right)^2+1}}\Bigg|_0^\infty = \boxed{\frac{\mu_0 I}{2\pi a} \otimes \frac{wb}{m^2}}$$

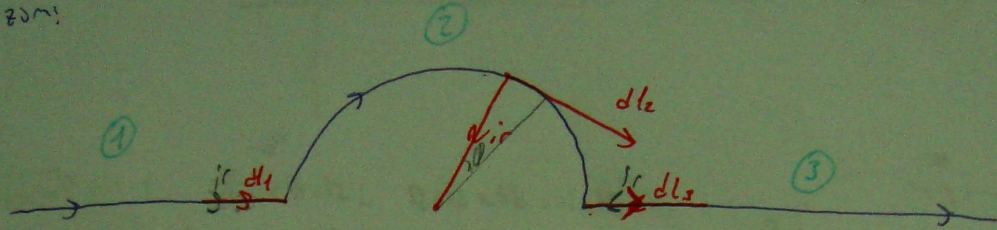
→ B'nin birimi $\frac{wb}{m^2}$ dir

Örnek 3:



P' noktasındaki Magnetik akı yoğunluğu vektörünü bulunuz

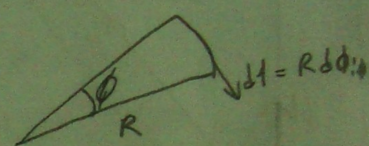
Çözüm:



① için: $dl_1 \times i_r = dl \times \sin 0 = 0$

② için: $dl_2 \times i_r = dl \times \sin 90 = dl$

③ için: $dl_3 \times i_r = dl \times \sin 180 = 0$



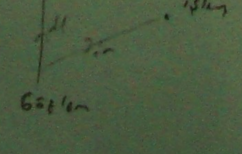
$$dB_T = dB_1 + dB_2 + dB_3 = 0 + dB_2 + 0 = dB_2 = \frac{\mu_0 I}{4\pi} \cdot \frac{dl_2 \times i_r}{r^2} = \frac{\mu_0 I}{4\pi} \frac{R.d\phi}{R^2}$$

$$dB_T = \frac{\mu_0 I}{4\pi} \frac{d\phi}{R} i_\phi$$

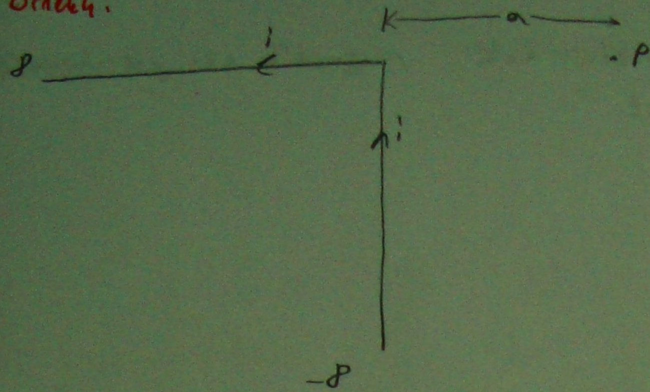
$$B = \frac{\mu_0 I}{4\pi R} \cdot \int_0^{\pi} \frac{d\phi}{R} \otimes = \frac{\mu_0 I}{4R} \quad , \quad \boxed{B = \frac{\mu_0 I}{4R} \otimes \frac{wb}{m^2}}$$

dl: her zaman akım yönünde

ir her zaman gözlem noktasından işlem noktasına



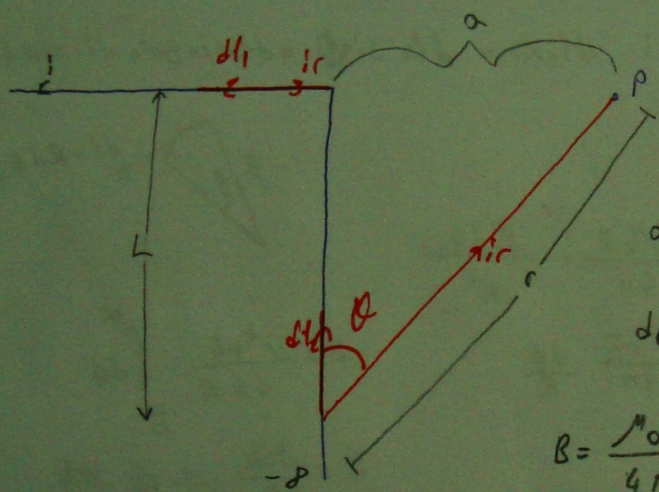
Örnek4:



→ P noktasındaki magnetik vektör potansiyelini bulun.

$$\left(\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{(a^2+x^2)}} \right)$$

Çözüm:



$dl_1 \times i_r = dl_1 \sin 180^\circ = 0$

$dl_2 \times i_r = dl_2 \times \sin\theta$

$$d_B = \frac{\mu_0 I}{4\pi} \cdot \frac{dl_2 \times i_r}{r^2} = \frac{\mu_0 I}{4\pi} \cdot \frac{a \; dl}{r^3}$$

$$d_B = \frac{\mu_0 I a}{4\pi r^3} dl$$

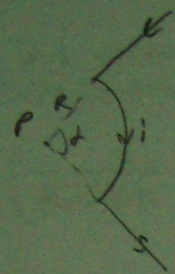
$$B = \frac{\mu_0 I a}{4\pi} \int_{-\infty}^{0} \frac{dl}{(a^2+L^2)^{3/2}} = \frac{\mu_0 I \cancel{a}}{4\pi} \cdot \frac{L}{a^2\sqrt{(a^2+L^2)}} \Bigg|_{-\infty}^{0}$$

$$= \frac{\mu_0 I}{4\pi a} \cdot \frac{1}{\sqrt{\left(\frac{a}{L}\right)^2+1}}$$

$r = \sqrt{a^2+L^2}$

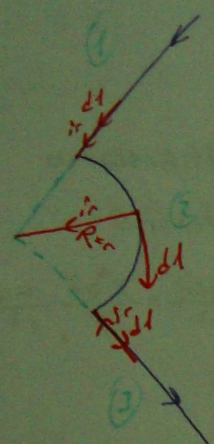
$$\boxed{B = \frac{\mu_0 I}{4\pi a} \otimes \frac{Wb}{m^2}}$$

Örnek5:



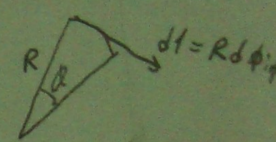
P noktasındaki magnetik akı yoğunluğunu (B) bulunuz.

Çözüm:



$dl_1 \times i_r = dl_3 \times i_r = 0$; $\sin 0 = \sin 180 = 0$

2 için : $dl_2 \times i_r = dl \times \sin\theta = dl \times \sin 90° = dl = R\,d\phi\, i_\phi$



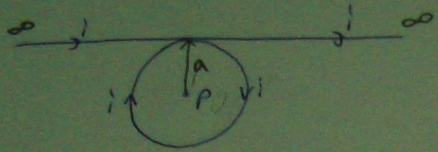
$$dB = \frac{\mu_0 I}{4\pi} \cdot \frac{R\,d\phi}{R^2}$$

$$= \frac{\mu_0 I}{4\pi} \cdot \frac{d\phi}{R}$$

$$B = \frac{\mu_0 I}{4\pi R} \cdot \int_0^\alpha d\phi$$

$$B = \frac{\mu_0 I}{4R} \cdot \frac{\alpha}{\pi} \otimes \frac{Wb}{m^2}$$

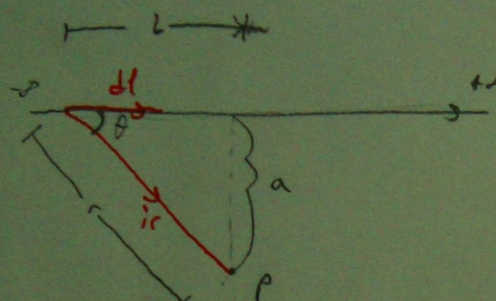
Örnek 6:



→ P noktasındaki magnetik akı yoğunluğunu bulunuz.

Çözüm

→ Süper pozisyon uygulanır.



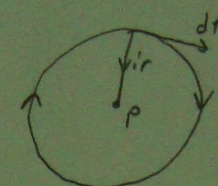
$dl \times i_r = dl \sin\theta$; $\sin\theta = \frac{a}{r}$; $r = \sqrt{\ }$

$$dB = \frac{\mu_0 I}{4\pi}\cdot\frac{dl \times i_r}{r^2} = \frac{\mu_0 I}{4\pi}\,\frac{a\,dl}{r^3}$$

$$= \frac{\mu_0 I}{4\pi}\cdot\frac{a}{(L^2+a^2)^{3/2}}\,dl$$

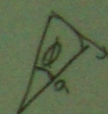
$$B = \frac{\mu_0 I a}{4\pi}\cdot 2\cdot\int_0^{\infty}\frac{dl}{(L^2+a^2)^{3/2}} = \frac{\mu_0 I \cancel{a}}{2\pi}\,\frac{L}{a^{\cancel{2}}\sqrt{a^2+L^2}}\Bigg|_0^{\infty}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi a} \otimes \frac{wb}{m^2}}$$



$dl \times i_r = dl \sin 90 = dl$

$$dB = \frac{\mu_0 I}{4\pi}\cdot\frac{dl}{r^2}$$



$dl = a\,d\phi\, i_\phi$

$$dB = \frac{\mu_0 I}{4\pi}\,\frac{\cancel{a}\,d\phi}{a^{\cancel{2}}} = \frac{\mu_0 I}{4\pi a}\,d\phi$$

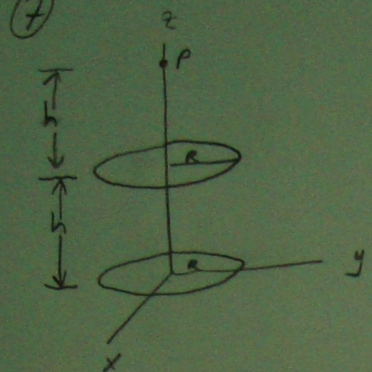
$$B = \frac{\mu_0 I}{4\pi a}\int_0^{2\pi} d\phi = \frac{\mu_0 I}{\cancel{4}\pi a}\,\cancel{2\pi} = \frac{\mu_0 I}{2a} \otimes \frac{wb}{m^2}$$

$$B_T = \frac{\mu_0 I}{2a}\left[\frac{1}{\pi} + 1\right] \otimes \frac{wb}{m^2}$$

dl ile ir arasındaki açı θ dır.

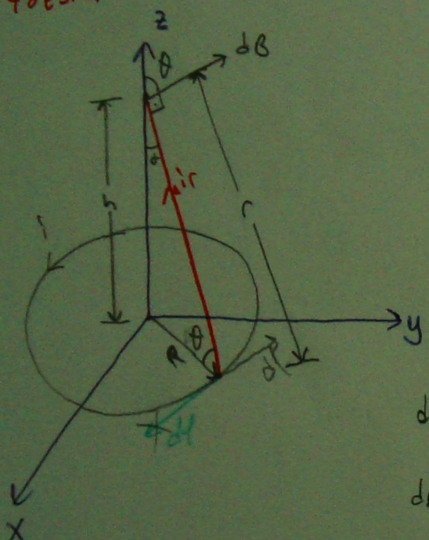
Örnek (2000 1. arasınav soru 3:)

7



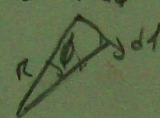
P noktasındaki magnetik endüksiyonu (B) bulunuz.

Çözüm:



→ dB tüm çember boyunca toplanırsa x, ve y bileşenleri birbirlerini nötrler ve elimizde sadece dBz bileşeni kalır. Bunu bulmak için dB yi cos θ ile çarparız

$dl = R d\phi$; $r = \sqrt{R^2+h^2}$; $\cos\theta = \frac{R}{r}$



$dl \times i_r = dl \cdot \sin 90 = dl$

$$dB = \frac{\mu_0 I}{4\pi} \cdot \frac{dl \times i_r}{r^2} = \frac{\mu_0 I}{4\pi} \cdot \frac{R\, d\phi\, i_\phi}{(R^2+h^2)^{1/2}}$$

$$dB_z = \frac{\mu_0 I}{4\pi} \cdot \frac{R^2\, d\phi}{(R^2+h^2)^{3/2}}$$

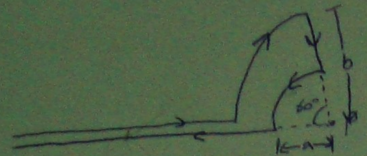
$$B_{z_1} = \frac{\mu_0 I}{4\pi} \frac{R^2}{(R^2+h^2)^{3/2}} \int_0^{2\pi} d\phi = \boxed{\frac{\mu_0 I}{2} \frac{R^2}{(R^2+h^2)^{3/2}}}$$

$$B_{z_2} = \frac{\mu_0 I R^2}{2\,(R^2+(2h)^2)^{3/2}}$$

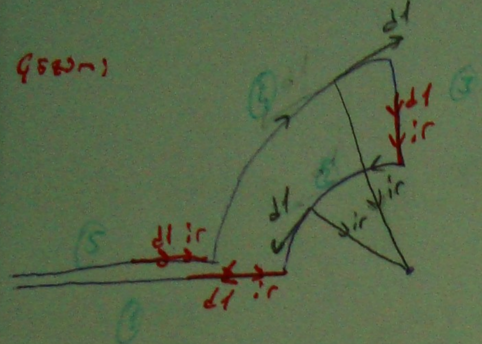
$$\boxed{B_T = \frac{\mu_0 I R^2}{2} \left[\frac{1}{(R^2+4h^2)^{3/2}} + \frac{1}{(R^2+h^2)^{3/2}} \right]}$$

Örnek8:

Biot-Savart denklemini kullanarak yanda verilen şekle göre P noktasındaki Magnetik akı yoğunluğu vektörünü (B) bulunuz.

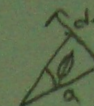


Çözüm:



①=③=⑤ ⇒ $dl \times i_r = dl \sin 180 = dl \sin 0 = dl \sin 0 = 0$

② ⇒ $dl \times i_r = dl \times \sin 90 = dl$



$dl = a\,d\phi\,(-i_\phi)$

④ ⇒ $dl \times i_r = dl \sin 90 = dl$

$dl = b\,d\phi\,(i_\phi)$

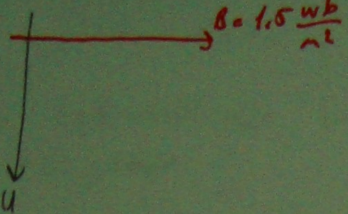
$$dB_2 = \frac{\mu_0 I}{4\pi}\cdot\frac{a\,d\phi}{a^2} = \frac{\mu_0 I}{4\pi a}\,d\phi \Rightarrow B_2 = \frac{\mu_0 I}{4\pi a}\int_0^{\pi/3} d\phi = \frac{\mu_0 I}{4\pi a}\cdot\frac{\pi}{3} = \boxed{\frac{\mu_0 I}{12a} \odot \frac{wb}{m^2}}$$

$$dB_4 = \frac{\mu_0 I}{4\pi}\cdot\frac{b\,d\phi}{b^2} = \frac{\mu_0 I}{4\pi b}\,d\phi \Rightarrow B_4 = \frac{\mu_0 I}{4\pi b}\int_0^{\pi/3} d\phi = \frac{\mu_0 I}{4\pi b}\,\frac{\pi}{3} = \frac{\mu_0 I}{12 b} \otimes \frac{wb}{m^2}$$

$$\boxed{B_T = \frac{\mu_0 I}{12}\left[\frac{1}{a} - \frac{1}{b}\right] \odot \frac{wb}{m^2}}$$

yakın olan telin B'si daha büyük.

Örnek 9:



B alanına dik olarak 5 mega elektron voltluk kinetik enerjiye sahip proton hareket ediyor. Proton üstüne etki eden kuvveti bulunuz. ($M_p = 1,67 \times 10^{-27}$ kg)

Çözüm

→ Öncelikle bir elektronun yükünün $= 1.6\times10^{-19}$ olduğunu bilmeniz gerekiyor.

$$ke = \underbrace{5\times10^{6}}_{5Me} \times \underbrace{1.6\times10^{-19}}_{1.\text{ elektronun yükü}} = 8\times10^{-13}\text{ Joule}$$

$$ke = \frac{1}{2} m v^2 \Rightarrow V = \sqrt{\frac{2ke}{m}} = \sqrt{\frac{2\times8\times10^{-13}}{1,67\times10^{-19}}} = 3,1\times10^{7}\ m/sn$$

0,1

0,1 1×10^{1}

$$F = eV\times B$$

$$F = 1,6\times10^{-19} \times 3,1\times10^{7} \times 1,5 = 7,4\times10^{-12} N$$

$$F = 7,4\times10^{-12} N$$

$$F = Q.E + Q.V\times B$$

Örnek11: $V = V_x i_x$ hızı ile hareket eden Q yükü $B = B_x i_x + B_y i_y + B_z i_z$ alanının içerisine giriyor. Hızı değişmeden geçebilmesi için $E = ?$

Çözüm:

Hız değişmemesi için ivme $= 0$ olmalıdır. ivmenin $= 0$ olması içinse $F = 0$ olması gerekir.

$F = EQ + Q\,V \times B$ dir.

Bu durumda

$F = 0 = EQ + Q\,V \times B \Rightarrow -E\cancel{Q} = \cancel{Q}\,V \times B \;;\; -E = V \times B$

$E = -V \times B$

$$V \times B = \begin{vmatrix} i_x & i_y & i_z \\ V_x & 0 & 0 \\ B_x & B_y & B_z \end{vmatrix} = 0\,i_x - (V_x B_z)\,i_y + (V_x B_y)\,i_z$$

$V \times B = -V_x B_z\, i_y + V_x B_y\, i_z$

$$\boxed{E = V_x B_z\, i_y - V_x B_y\, i_z}$$

Örnek10: $B=\frac{\mu_0}{4\pi}\iiint\frac{J\times i_r}{r^2}\,d\tau$ denkleminden başlayarak, B'nin divergence'inin sıfır $(\nabla\cdot B=0)$ olduğunu gösteriniz. Bu ne anlama gelir kısaca açıklayınız.

$[\nabla\cdot(A\times B)=B(\nabla\times A)-A(\nabla\times B)]$

Çözüm:

$$B=\frac{\mu_0}{4\pi}\iiint\frac{J\times i_r}{r^2}\,d\tau \quad \longrightarrow \text{ İki tarafında Div ini alalım}$$

$$\nabla\cdot B=\nabla\cdot\frac{\mu_0}{4\pi}\int\frac{J\times i_r}{r^2}\,d\tau \quad ; \quad \nabla\cdot(A\times B)=B.(\nabla\times A)-A(\nabla\times B)$$

$$\nabla\cdot\left(\frac{J\times i_r}{r^2}\right)=\nabla\cdot\left(J\times\frac{i_r}{r^2}\right)=\frac{i_r}{r^2}(\nabla\times J)-J\left(\nabla\times\frac{i_r}{r^2}\right)$$

$$\left(\frac{\partial}{\partial x}\right) \quad (x',y',z')$$

$$\nabla\left(\frac{1}{r}\right)=-\frac{i_r}{r^2} \quad , \quad \boxed{\nabla\cdot B=0}$$

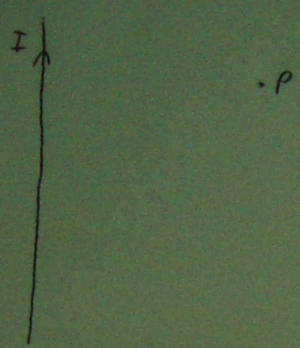
→ Ayrıca Gradientin Curl'ü $=0$ dır. $\left(\nabla\times\nabla\left(\frac{1}{r}\right)=0\right.$

Kanıtı:

$$\nabla\times\frac{i_r}{r^2}=\nabla\times\frac{\bar{r}}{r^3}=\begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x-x'}{r^3} & \frac{y-y'}{r^3} & \frac{z-z'}{r^3}\end{vmatrix}=0$$

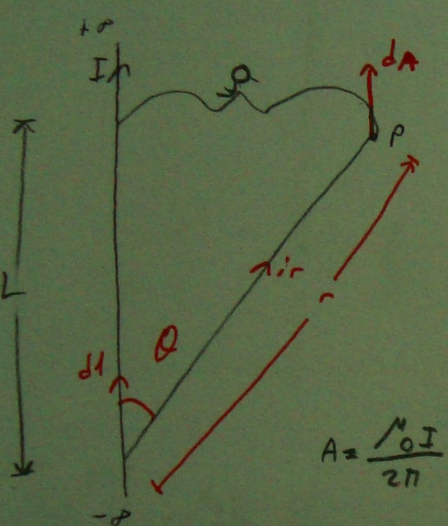
$$\iiint(\nabla\cdot B)\,d\tau=\oiint B.da=0$$

Örnek12:



→ Magnetik Vektör potansiyelini(A) bulunuz.

Çözüm:



$$dA = \frac{\mu_0 I}{4\pi}\cdot\frac{dl}{r} \quad ; \quad r=\sqrt{\rho^2+L^2}$$

$$dA = \frac{\mu_0 I}{4\pi}\cdot\frac{dl}{\sqrt{\rho^2+L^2}}$$

$$A = \frac{\mu_0 I}{4\pi}\int_{-L}^{+L}\frac{dl}{(\rho^2+L^2)^{1/2}} = \frac{\mu_0 I}{4\pi}\cdot 2\cdot\int_{0}^{L}\frac{1}{(\rho^2+L^2)^{1/2}}\,dl$$

$$A = \frac{\mu_0 I}{2\pi}\left\{\ln\left[L+\sqrt{L^2+\rho^2}\right]-\ln\rho\right\} = \frac{\mu_0 I}{2\pi}\ln\left[\frac{L+\sqrt{L^2+\rho^2}}{\rho}\right]$$

$L\to\infty$, $A\to\infty$

$L>>\rho$ ise $\quad L^2+\rho^2\approx L^2$

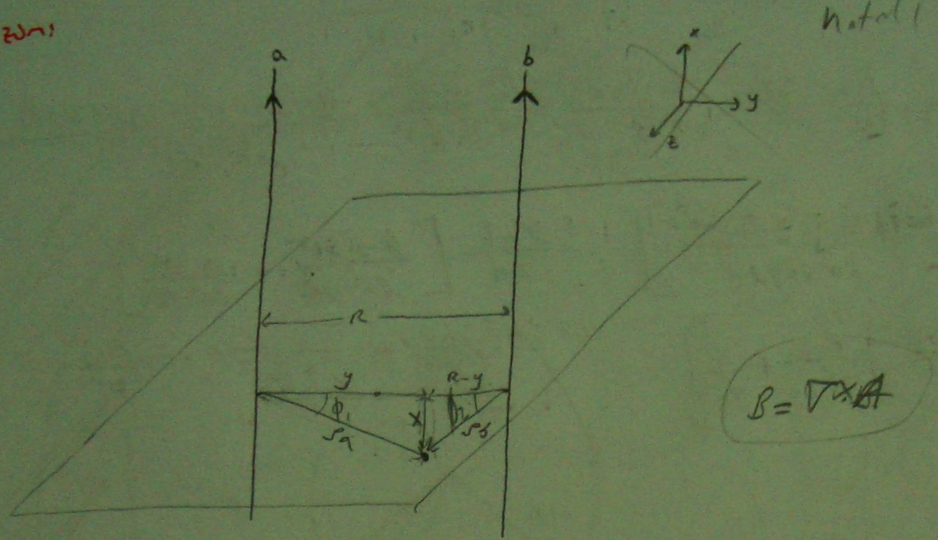
$$\boxed{A = \frac{\mu_0 I}{2\pi}\ln\frac{2L}{\rho}\,i_z}\ \frac{Wb}{m}$$

Örnek 13:

iki iletken arasında b'ye yakın noktada magnetik vektör potansiyeli (A) nedir?

Çözüm:

noktalı



$$\boxed{B = \nabla \times A}$$

a'ya göre

$$d_A = \frac{\mu_0 I}{4\pi}\frac{dl}{\rho_a} = \frac{\mu_0 I}{4\pi}\cdot\frac{dl}{\sqrt{y^2+L^2}}$$

$$A = \frac{\mu_0 I}{4\pi}\cdot 2\int_0^L \frac{1}{(y^2+L^2)^{1/2}}\,dl = \frac{\mu_0 I}{2\pi}\cdot Ln\,\frac{L+\sqrt{y^2+L^2}}{\rho_a}$$

$L \gg y \Rightarrow$

$$A_a = \frac{\mu_0 I}{2\pi} Ln\,\frac{2L}{\rho_a}\, i_z$$

b'ye göre

$$d_A = \frac{\mu_0 I}{4\pi}\cdot\frac{dl}{r} = \frac{\mu_0 I}{4\pi}\frac{dl}{\rho_b} = \frac{\mu_0 I}{4\pi}\frac{dl}{\left[(R-y)^2+L^2\right]^{1/2}}$$

$$A = \frac{\mu_0 I}{2\pi}\int_0^L \frac{1}{\left[(R-y)^2+L^2\right]^{1/2}}\,dl$$

$L \gg \rho_b \Rightarrow$

$$A = \frac{\mu_0 I}{2\pi} Ln\,\frac{L+\left[(R-y)^2+L^2\right]^{1/2}}{\rho_b}$$

$ln\,\frac{A}{b}$

$$A_b = \frac{\mu_0 I}{2\pi} Ln\,\frac{2L}{\rho_b}\, -i_z\,\frac{wb}{m}$$

$$A_T = A_a - A_b = \frac{\mu_0 I}{2\pi} Ln\left(\frac{\cancel{2L}}{\rho_a}\cdot\frac{\rho_b}{\cancel{2L}}\right)$$

$$A_T = \frac{\mu_0 I}{2\pi} Ln\,\frac{\rho_b}{\rho_a}\,\odot\,\frac{wb}{m}$$

Ama bize ρ_a ve ρ_b verilmemiş x y ve R li çözmemiz lazım

$$\boxed{A_T = \frac{\mu_0 I}{4\pi} Ln\left(\frac{x^2+(R-y)^2}{y^2+x^2}\right) i_z\,\frac{wb}{m}}$$

Aynı soruda magnetik vektör potansiyelinden (A), magnetik akı yoğunluğunu (B) bulunuz.

$$B=\nabla\times A=\begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & A_z \end{vmatrix} = \underbrace{i_x\left(\frac{\partial}{\partial y}A_z\right)}_{B_x} - \underbrace{i_y\left(\frac{\partial}{\partial x}A_z\right)}_{B_y} + i_z(0)$$

$$B_x=\frac{\partial}{\partial y}A_z=\frac{\partial}{\partial y}\cdot\frac{\mu_0 I}{4\pi}\ln\left(\frac{(R-y)^2+x^2}{x^2+y^2}\right)=\frac{\mu_0 I}{4\pi}\cdot\frac{\cancel{x^2+y^2}}{(R-y)^2+x^2}\cdot\frac{-2(R-y)(x^2+y^2)-2y\left((R-y)^2+x^2\right)}{(x^2+y^2)^2}$$

$$B_x=\frac{\mu_0 I}{2\pi}\left[\frac{-(R-y)\rho_b^2-y\rho_a^2}{\rho_a^2\rho_b^2}\right]=-\frac{\mu_0 I}{2\pi}\left[\frac{(R-y)\cancel{\rho_b^2}}{\rho_a^2\cancel{\rho_b^2}}+\frac{y\cancel{\rho_a^2}}{\cancel{\rho_a^2}\rho_b^2}\right]$$

$$B_x=-\frac{\mu_0 I}{2\pi}\left[\frac{R-y}{\rho_a^2}+\frac{y}{\rho_b^2}\right]$$

$$B_y=\frac{\partial}{\partial x}A_z=\frac{\partial}{\partial x}\cdot\frac{\mu_0 I}{4\pi}\cdot Ln\left(\frac{(R-y)^2+x^2}{x^2+y^2}\right)=\frac{\mu_0 I}{4\pi}\cdot\frac{\cancel{x^2+y^2}}{(R-y)^2+x^2}\cdot\frac{2x(x^2+y^2)-2x\left((R-y)^2+x^2\right)}{(x^2+y^2)^2}$$

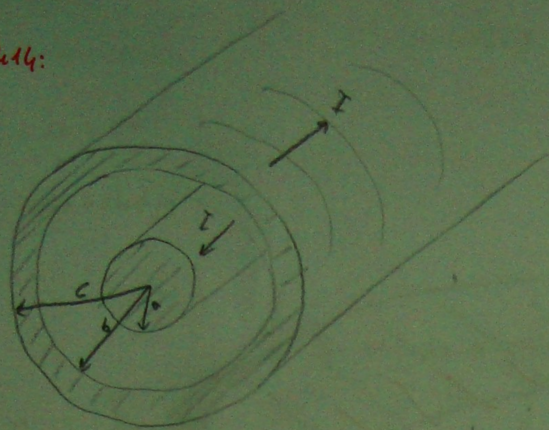
$$B_y=\frac{\mu_0 I}{4\pi}\cdot\frac{2x\rho_b^2-2x\rho_a^2}{\rho_b^2\,\rho_a^2}=\frac{\mu_0 I}{2\pi}\cdot\left[\frac{x\cancel{\rho_b^2}}{\cancel{\rho_b^2}\rho_a^2}-\frac{x\rho_a^2}{\rho_b^2\rho_a^2}\right]=\frac{\mu_0 I}{2\pi}\left(\frac{x}{\rho_a^2}-\frac{x}{\rho_b^2}\right)$$

$$B_y=-\frac{\mu_0 I}{2\pi}\left[\frac{x}{\rho_b^2}-\frac{x}{\rho_a^2}\right]$$

iki telin tam ortasında $x=0$, $y=\frac{R}{2}$.

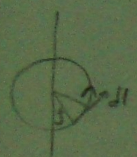
$B_y=0$, $B_x=-\frac{2\mu_0 I}{\pi R}$

Örnek 14:



Soru: Tüm bölgelerde magnetik akı yoğunluğunu (B) bulunuz

Çözüm: Soru bir Amper yasası sorusudur, önce B 'den $\oint B.dl$'yi bulalım



$B=\frac{\mu_0 I}{2\pi\rho} i_\phi$; $dl=\rho\, d\phi\, i_\phi$ ⇒

$\oint B.dl = \int \frac{\mu_0 I}{2\pi\rho} i_\phi\, dl$ ⇒ $\oint_C B.dl = \int \frac{\mu_0 I}{2\pi\cancel{\rho}} \overbrace{i_\phi\, \cancel{\rho}\, d\phi\, i_\phi}^{1} = \int \frac{\mu_0 I}{2\pi} d\phi = \frac{\mu_0 I}{2\pi}.2\pi$

$\boxed{\oint B dl = \mu_0 I = \mu_0 J \pi r^2}$

Şimdi soruyu 4 ayrı bölge için çözelim.

$r<a$, $a<r<b$, $b<r<c$, $c<r$

şimdi so

r<a için:

$\oint B dl = \mu_0 I_{net}$

$2\pi r B = \mu_0 I . \frac{\pi r^2}{\pi a^2}$

$\boxed{B = \frac{\mu_0 I r}{2\pi a^2} i_\phi}$

a<r<b

$\oint B dl = \mu_0 I_{net}$

$2\pi r B = \mu_0 I.$

$\boxed{B = \frac{\mu_0 I}{2\pi r} i_\phi}$

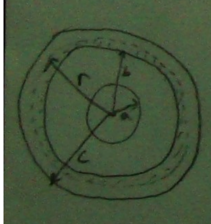
r>c için

r>c için net akı = 0 dır. Bu durumda B=0 olur.

$\oint B dl = \mu_0 I_{net} = \mu_0 (I - I)$

$B = 0$

c>r>b için



$\oint B dl = \mu_0 I_{net} = \mu_0 \left(I - (J\pi r^2 - J\pi b^2)\right)$

Toplam akım = $I = (\pi c^2 - \pi b^2) J$

$\oint B dl = \mu_0 \left[J\pi(c^2-b^2) - J\pi(r^2-b^2)\right]$

$= \mu_0 J\pi (c^2 - r^2)$; $J\pi = \frac{I}{c^2-b^2}$

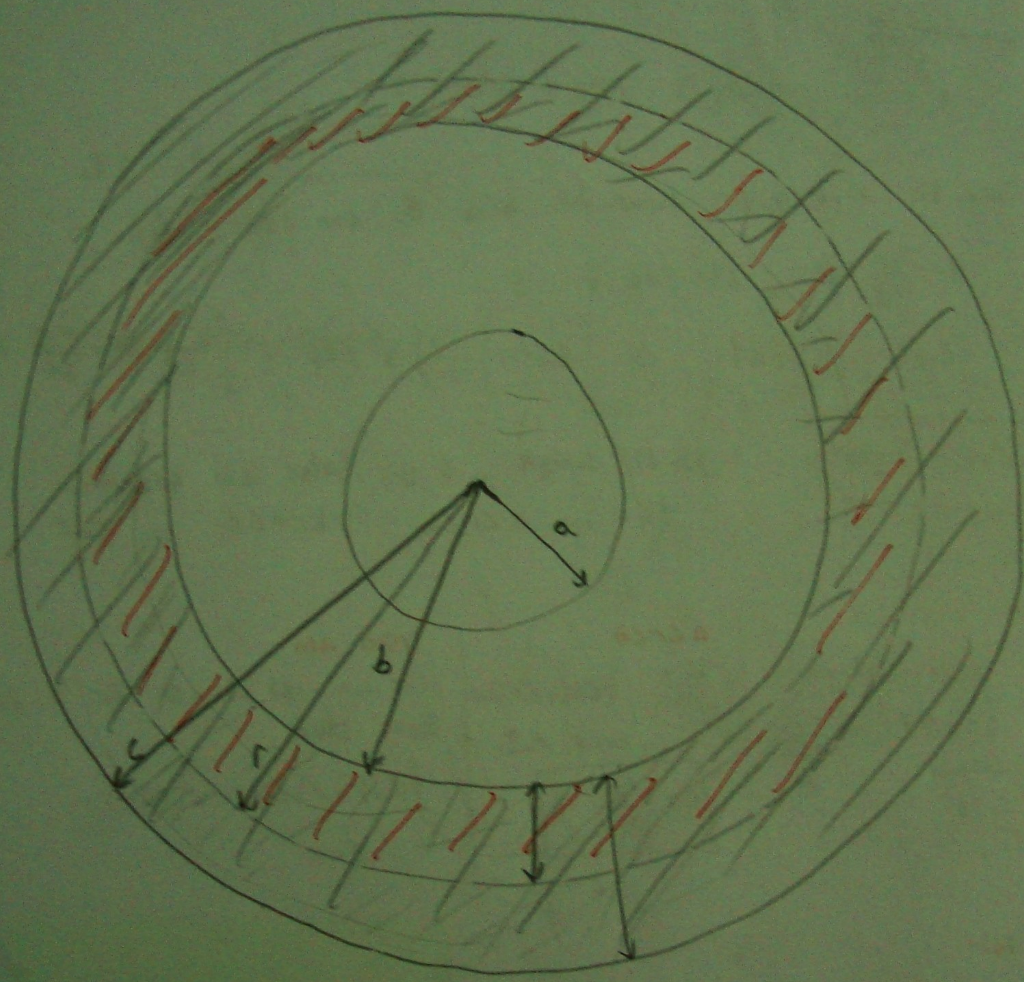
$2\pi r B = \mu_0 I \left(\frac{c^2-r^2}{c^2-b^2}\right) i_\phi$

$\boxed{B = \frac{\mu_0 I}{2\pi r} \left(\frac{c^2-r^2}{c^2-b^2}\right) i_\phi}$

$B = \frac{\mu_0 I}{2\pi r} \left(\frac{r^2-b^2}{c^2-b^2}\right) i_\phi$

olması gerekmez mi?

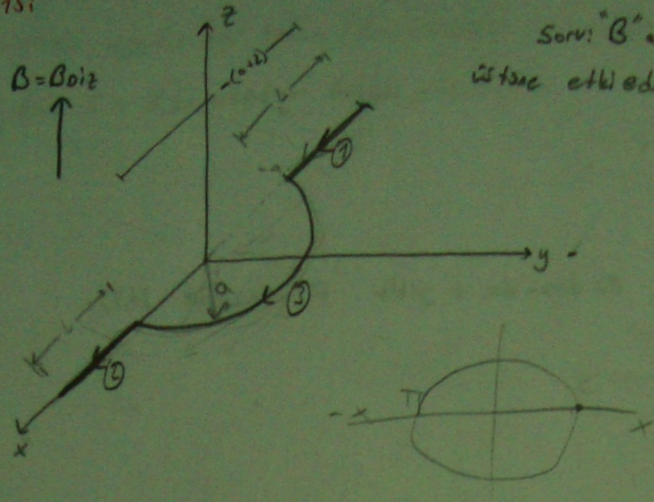
$c > r > b$

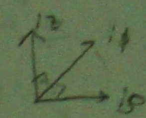


$$\frac{\text{Kırmızıdan geçen akım}}{\text{Siyahtan geçen akım}} = \frac{J\pi(r^2-b^2)}{J\pi(c^2-b^2)}$$

Örnek 15:

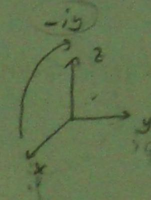
Soru: "B" alanının içinde kalan iletkenin üstüne etki eden toplam kuvveti bulunuz.

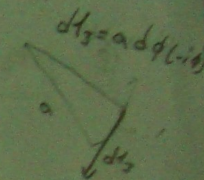




Çözüm:

$d\vec{F} = I\, d\vec{l} \times \vec{B}$





1 nolu bölge için: $dl_1 = dx\, i_x$, $B = B_0 i_z$, $I = I$

$dF_1 = I\, \overbrace{dx\, i_x \times B_0 i_z}^{-i_y} = I\, dx\, B_0(-i_y)$

$dF_1 = I\, dx B_0(-i_y) \Rightarrow \quad F_1 = \int I B_0(-i_y)\, dx = I B_0(-i_y) \int_{-(a+L)}^{-a} dx = I B_0(-i_y)\left(-\cancel{a} + \cancel{a} + L\right)$

$$\boxed{F_1 = -B_0 I L\, i_y}$$

2 nolu bölge için: $dF = I\, d\vec{l} \times \vec{B}$; $dl_2 = dx\, i_x$; $B = B_0 i_z$; $I = I$

$dF_2 = I\, \overbrace{dx\, i_x \times B_0 i_z}^{-i_y} = B_0 I\, dx(-i_y)$

$dF_2 = B_0 I\, dx(-i_y) \Rightarrow \quad F_2 = \int B_0 I\, dx(-i_y) = B_0 I(-i_y) \int_a^{a+L} dx = B_0 I(-i_y)\left(\cancel{a} + L - \cancel{a}\right)$

$$\boxed{F_2 = -B_0 I L\, i_y}$$

3 nolu bölge için: $dF = I\, d\vec{l}_3 \times \vec{B}$; $dl = a\, d\phi(-i_\phi)$; $B = B_0 i_z$; $I = I$

$dF_3 = I \cdot a\, d\phi(-i_\phi) \times B_0 i_z = I a\, d\phi\, B_0\, i_\rho$

$dF_3 = I a\, d\phi\, B_0 i_\rho \Rightarrow \quad F_3 = \int_\pi^0 I a\, d\phi\, B_0 i_\rho = -I a B_0 \int_0^\pi i_\rho\, d\phi$; $i_\rho = i_x \cos\phi + i_y \sin\phi$

$F_3 = -B_0 I a \int_0^\pi (\cancel{\cos\phi\, i_x} + \sin\phi\, i_y)\, d\phi = -B_0 I a\left[-\cos\phi\, i_y\right]_0^\pi = -B_0 I a\, i_y(\underbrace{-\cos\pi}_{-(-1)} + \underbrace{\cos 0}_{1}) = -2B_0 I a\, i_y$

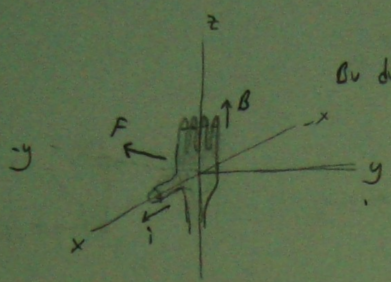
$F_3 = -2B_0 I a\, i_y$

$F_T = F_1 + F_2 + F_3 = -2B_0 I L\, i_y - 2B_0 I a\, i_y = -2B_0 I(L+a)\, i_y$

$\int \cos x \, dx = -\sin x$ $\qquad$ $dF = I \, d\bar{l} \times \bar{B}$

$\int \sin x \, dx = \cos x$

$i_\phi = \cos\phi \, \underline{i_x} + \sin\phi \, \underline{i_y}$



Bu durumda + yükler F yönünde itilir

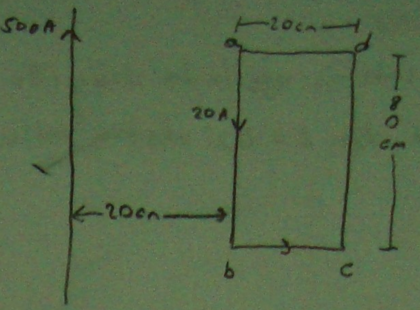
iş integral dışına çıkamaz

3 nolu BÖLGE

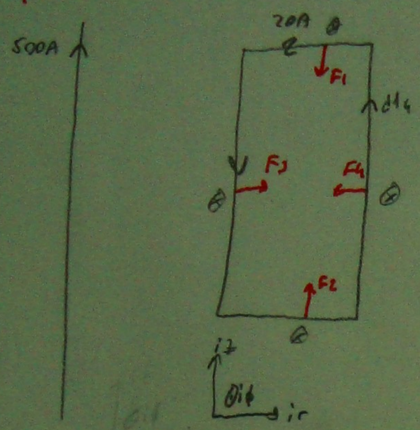
× GELİYOR

Örnek16: Quiz 1. örgn:

Şekilde görülen çok uzun telden 500A akım geçmektedir. Üzerinden 20A akım geçen 80x20cm ebatlarında ve uzun tele paralel tek sarımlı dikdörtgen bobin üzerindeki toplam kuvveti bulunuz.



Çözüm:

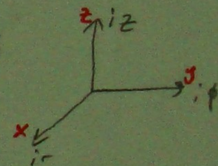


$\oint B\,dl = \mu_0 I$

$2\pi r B = \mu_0 I$

$\boxed{B = \frac{\mu_0 I}{2\pi r}\, i_\phi}$

$dF = I\, dl \times B$



F_1 için: $dF_1 = I\, dl_1 \times B$; $dl_1 = dr_{(-i_r)} = -dr\, i_r$;

$dF_1 = -I\,dr\, i_r \times \frac{\mu_0 I}{2\pi r} i_\phi \Rightarrow F_1 = -I \frac{\mu_0 I\, i_z}{2\pi} \int_{0,4}^{0,2} \frac{1}{r}\, dr$

$i_r \times i_\phi = i_z$

$\boxed{F_1 = \frac{-20 \cdot \mu_0 \cdot 500}{2\pi} \cdot \ln \frac{0,2}{0,4}\; i_z}$

$|F_1| = |F_2|$

F_1 ve F_2 birbirini söndürler ve toplamları "0" olur hesaplamaya gerek yoktur.

F_3 için: $dF_3 = I\, dl_3 \times B$; $dl_3 = dl_{(-i_z)} = -dl\, i_z$

$dF_3 = -I\, dl\, i_z \times \frac{\mu_0 I}{2\pi r} i_\phi = +I\, dl \frac{\mu_0 I}{2\pi r} i_r$

$i_z \times i_\phi = -i_r$

$F_3 = +I \frac{\mu_0 I\, i_r}{2\pi r} \int_{80}^{0} dl = +\frac{20 \times \mu_0 \times 500}{2\pi \times 0,2} \cdot -0,8$

$F_3 = 18 \times 10^{-3}\, N$

F_4 için: $dF_4 = I\, dl_4 \times B$; $dl_4 = dl\, i_z$

$dF_4 = I\, dl\, i_z \times \frac{\mu_0 I}{2\pi r} i_\phi = -I\, dl \frac{\mu_0 I}{2\pi r} i_r$

$i_z \times i_\phi = -i_r$

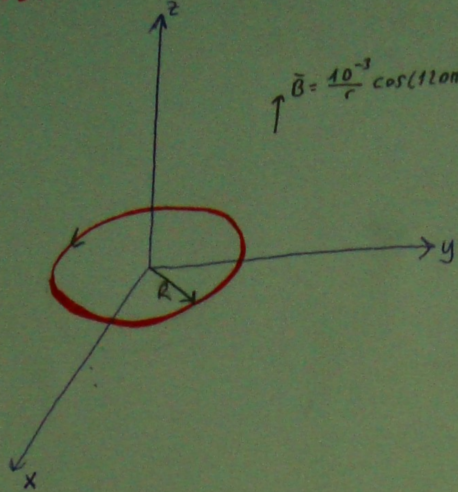
$F_4 = -I \frac{\mu_0 I}{2\pi r} i_r \int_{0}^{80} dl$

$F_4 = \frac{-20\, \mu_0\, 500}{2\pi \cdot 0,4} \cdot 0,8$

$F_4 = -4 \times 10^{-3}$

$\boxed{F_T = 4 \times 10^{-3}\ \text{Newton}}$

Örnek 17:



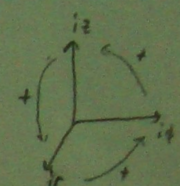
$\bar{B} = \frac{10^{-3}}{r}\cos(120\pi t)\, i_z \ \frac{Wb}{m^2}$

a) indüklenen Emk

b) E_ϕ

c) Telin direnci $10\,\Omega$ ise $I=?$

Çözüm:



a) $\boxed{Emk = \oint E\cdot dl = \oint (u\times B)\cdot dl + \frac{-d}{dt}\iint \bar{B}\cdot d\bar{a}} = \oint (u\times B)\cdot dl + \frac{-d\phi_B}{dt}$

Herhangi bir hız sözkonusu olmadığı için $Emk = \oint E\cdot dl = \frac{-d}{dt}\iint B\cdot da$

$Emk = \oint E\cdot dl = \frac{-d}{dt}\iint \bar{B}\cdot da = \frac{-d}{dt}\iint \frac{10^{-3}}{r}\cos(120\pi t)\, i_z\cdot da$

$da = r\,dr\,d\phi\, i_z$

$Emk = \frac{-d}{dt}\iint \frac{10^{-3}}{\cancel{r}}\cos(120\pi t)\, i_z\cdot \cancel{r}\,dr\,d\phi\, i_z = \frac{-d}{dt}\int_0^R\int_0^{2\pi} 10^{-3}\cos(120\pi t)\,dr\,d\phi$

$i_z\cdot i_z = 1$

$= 10^{-3}\cdot \frac{-d}{dt}\cos(120\pi t)\underbrace{\int_0^R\int_0^{2\pi} dr\,d\phi}_{2\pi R} = 10^{-3}\sin(120\pi t)\cdot(120\pi)\cdot 2\pi R$

$Emk = (2\pi R)\cdot 10^{-3}\cdot \sin(120\pi t)\cdot(120\pi)$ Volt

c) $i = \frac{V}{R} = \frac{2\pi R\cdot 10^{-3}\cdot \sin(120\pi t)\cdot 120\pi}{10} = 24\pi^2 R\sin(120\pi t)\times 10^{-3}$ Amper

$= 24\pi^2 R\sin(120\pi t)$ mA

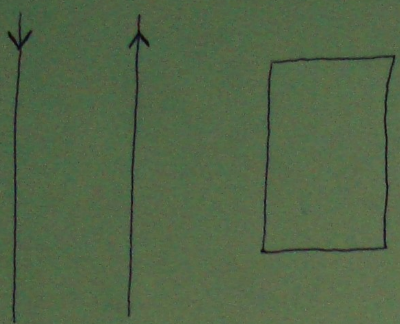
b) Elektrik alanını bulalım:

$\underbrace{\oint E\,dl}_{2\pi r E_\phi} = Emk$; $\cancel{2\pi R}\cdot E_\phi = (\cancel{2\pi R})10^{-3}\cdot 120\pi\sin(120\pi t)$

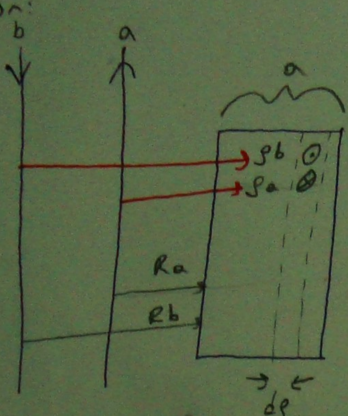
$\boxed{E_\phi = 12\pi\sin(120\pi t)\ \frac{mV}{m}}$

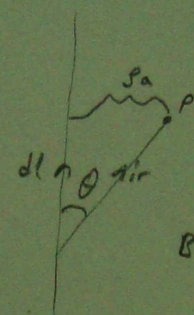
Örnek 23:

Emk=?



Çözüm:





$$dB = \frac{\mu_0 I}{4\pi} \cdot \frac{dl \times a_r}{r^2}$$

$$|dl \times a_r| = dl \sin\theta = \frac{\rho_a}{r} \cdot dl$$

$$B = \frac{\mu_0 I \rho_a}{4\pi} \int_{-\infty}^{\infty} \frac{1}{r} \, dl$$

$$B = \frac{\mu_0 I \rho_a}{4\pi} \cdot 2 \cdot \int_0^{\infty} \frac{1}{\sqrt{l^2+\rho_a^2}} \, dl = \frac{\mu_0 I}{2\pi \rho_a} \cdot \frac{1}{\sqrt{\left(\frac{l^2}{\rho^2}\right)+1}} \Bigg|_0^{\infty} \Rightarrow B_a = \frac{\mu_0 I}{2\pi \rho_a}$$

$$B_T = B_a + B_b = \frac{\mu_0 I}{2\pi} \left[\frac{1}{\rho_a} - \frac{1}{\rho_b}\right] \otimes \frac{wb}{m^2} \qquad da = h\,d\rho$$

Dikdörtgen bobini kesen toplam akı: Φ

$$\Phi = \iint \bar{B} \cdot da = \int \frac{\mu_0 I}{2\pi} \left[\int_{R_a}^{R_a+a} \frac{d\rho_a}{\rho_a} - \int_{R_b}^{R_b+a} \frac{d\rho_b}{\rho_b} \right] da = \frac{\mu_0 I h}{2\pi} \left\{ Ln \frac{R_a+a}{R_a} - Ln \frac{R_b+a}{R_b} \right\}$$

$$\Phi = \frac{\mu_0 I h}{2\pi} Ln \frac{R_b(a+R_a)}{R_a(a+R_b)}$$

$$emk = \oint E \cdot dl = -\frac{d\phi}{dt}$$

$$Emk = \frac{\mu_0 h}{2\pi} Ln \frac{R_b(a+R_a)}{R_a(a+R_b)} \cdot \frac{dI}{dt}$$